

MICROECONOMICS 2

Week 1: Consumer Utility and Choice

Course alignment: ECON2101 (ANU) — intermediate microeconomics, Week 1 (Consumer Utility and Choice; Nicholson & Snyder Ch. 3, background Chs. 1–2)

Prerequisites: Micro 101 Ch. 4 (Consumer Theory); partial derivatives

Chapter covers: preference axioms and utility representation, monotonic transformations, marginal utility and MRS by calculus, the four standard utility functions, budget constraints, interior optima (tangency and equal bang-per-buck), corner and kink solutions, two full worked optimisations, key propositions, common mistakes, practice problems

1. From Micro 1 to Micro 2

Micro 101 described consumer choice with diagrams; Micro 2 gives it an engine. The pictures stay the same — indifference curves meeting budget lines — but every slope now has a formula, every optimum is solved rather than pointed at, and “the consumer picks the best affordable bundle” becomes a two-equation system you can answer numerically. Everything later in ECON2101 (demand, comparative statics, welfare, general equilibrium) runs on this week's machinery.

2. Preferences and the Axioms

We start from preferences, not utility. A consumer compares bundles (x_1, x_2) , and we impose standard assumptions on those comparisons — the same four you met in Micro 101, now doing real mathematical work:

Assumption	Meaning	Implication
Completeness	Any two bundles can be ranked (or the consumer is indifferent).	Preferences are defined over all bundles — no “can't compare.”
Transitivity	If $A \succcurlyeq B$ and $B \succcurlyeq C$, then $A \succcurlyeq C$.	Rankings chain consistently; no cycles.
Monotonicity	More of a good, holding the other fixed, is weakly better.	Indifference curves slope downward.
Convexity	Averages are at least as good as extremes.	Curves bow toward the origin — diminishing MRS.

Theorem — Utility Representation

If preferences satisfy completeness, transitivity, continuity, and monotonicity, there exists a utility function $U(x_1, x_2)$ representing them: $U(A) \geq U(B)$ exactly when $A \succsim B$. This is the bridge from psychology to mathematics — from here on we work with U directly.

Ordinal Utility and Monotonic Transformations

Utility is **ordinal**: only rankings matter, not magnitudes — race positions, not race times. Any increasing transformation of U (squaring, logging, doubling) preserves the ranking and therefore represents the *same preferences*. $U = x_1x_2$, $U = x_1^2x_2^2$, and $U = \ln x_1 + \ln x_2$ describe the same consumer, share the same indifference curves, and generate the same demand. Exam use: you may replace U with any monotonic transformation to make differentiation easier — a log transform before differentiating a Cobb–Douglas is the classic time-saver.

3. Marginal Utility and the MRS

For differentiable $U(x_1, x_2)$, **marginal utility** is the partial derivative — the rate utility rises with a small increase in one good, holding the other fixed:

Marginal Utility

$$MU_1 = \partial U / \partial x_1 \quad MU_2 = \partial U / \partial x_2$$

Partial derivatives of the utility function. Under diminishing marginal utility, each additional unit of a good adds less utility than the one before.

The **marginal rate of substitution** is the rate the consumer will trade good 2 for one more unit of good 1 while staying on the same indifference curve — the (absolute) slope of the curve:

MRS — Definition and Formula

$$MRS_{12} = -(dx_2/dx_1)|U \text{ constant} = MU_1 / MU_2$$

The good you're **gaining goes on top**: MU_1 leads because x_1 is what's being acquired.

Diminishing MRS — the ratio falling as x_1 rises along a curve — is what makes indifference curves convex, and it's your sign-error check: if your computed MRS grows along the curve for a standard utility function, revisit your partials.

Calculation — MRS of $U = x_1^2 x_2$ at (2, 5)

$$MU_1 = 2x_1x_2 = 20; \quad MU_2 = x_1^2 = 4.$$

$MRS = 2x_1x_2 \div x_1^2 = 2x_2/x_1$ — always simplify the ratio before substituting numbers.

At (2, 5): $MRS = 2(5)/2 = 5$ — the consumer here would give up to 5 units of good 2 for one more unit of good 1.

4. The Utility Function Zoo

Four functional forms cover nearly every ECON2101 problem. Identifying the form is always step one, because it determines the solution method in section 6 — tangency only works for the smooth cases.

The Utility Function Zoo — Four Shapes, Four Solution Methods

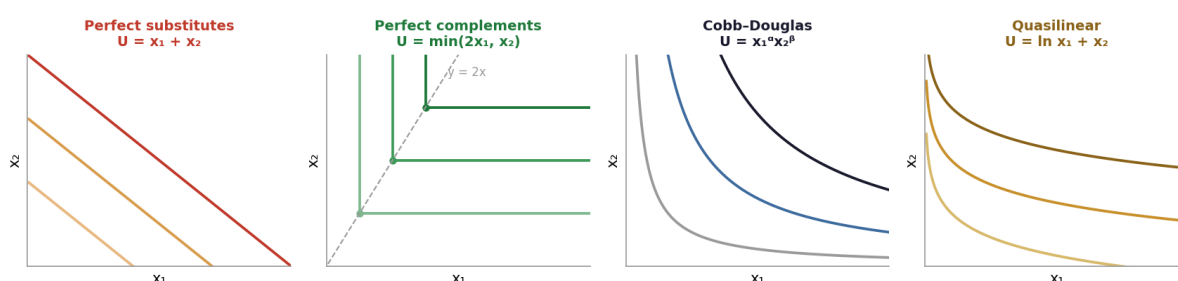


Figure 1.1 — The four standard utility functions. Straight lines (substitutes), right angles (complements), smooth convex curves (Cobb–Douglas), and vertically shifted curves (quasilinear).

Utility function	Formula and MRS	Key feature
Perfect substitutes	$U = ax_1 + bx_2$; $MRS = a/b$ (constant)	Straight-line curves. Consumer cares only about totals — optimum is a corner (section 7).
Perfect complements	$U = \min(ax_1, bx_2)$; MRS undefined at the kink	L-shaped curves; goods consumed in fixed proportions. Solve with the kink condition $ax_1 = bx_2$.
Cobb–Douglas	$U = x_1^\alpha x_2^\beta$; $MRS = (\alpha/\beta)(x_2/x_1)$	Smooth, convex — the exam workhorse. Interior optimum by tangency.
Quasilinear	$U = v(x_1) + x_2$; $MRS = v'(x_1)$ — depends only on x_1	Curves are vertical shifts of one another. Demand for x_1 is independent of income — a quirk that returns in welfare analysis.

5. The Budget Constraint

Preferences say what the consumer *wants*; the budget says what they *can afford*. With income m and prices p_1, p_2 :

Budget Constraint and Budget Line

$$p_1x_1 + p_2x_2 \leq m \quad (\text{line: } p_1x_1 + p_2x_2 = m)$$

Slope = $-p_1/p_2$ — the market's exchange rate between the goods, as opposed to the MRS, the consumer's. Intercepts m/p_1 and m/p_2 . Income changes shift the line in parallel; a price change pivots it around the other good's intercept.

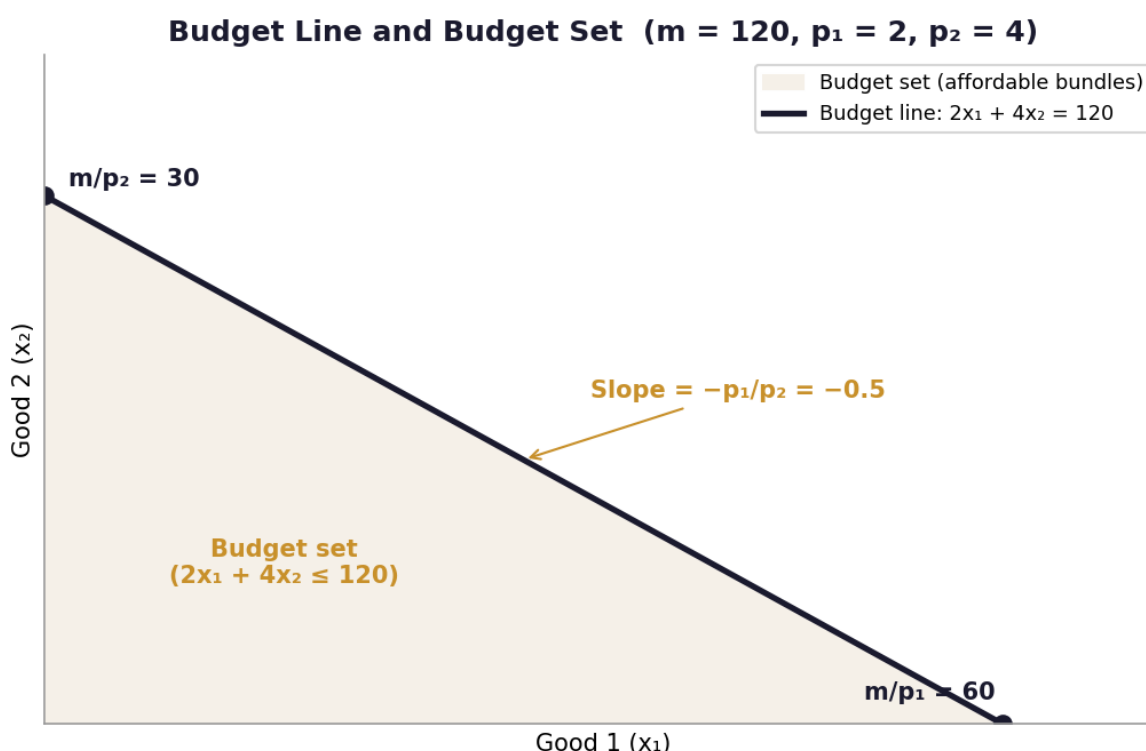


Figure 1.2 — The budget line for $m = 120, p_1 = 2, p_2 = 4$ (the worked example of section 6). Shaded region: the budget set.

6. Optimal Choice — Interior Solutions

The consumer's problem: maximise $U(x_1, x_2)$ subject to $p_1x_1 + p_2x_2 = m$. For smooth convex preferences the optimum is a **tangency** — the highest indifference curve just touching the budget line:

Interior Optimum — Two Conditions

$$MRS_{12} = p_1/p_2 \quad \text{and} \quad p_1x_1^* + p_2x_2^* = m$$

Two equations, two unknowns. The tangency can be rewritten as $MU_1/p_1 = MU_2/p_2$ — the **equal bang-per-buck rule**: at the optimum, the last dollar spent on each good buys the same marginal utility. If bang-per-buck were unequal, shifting a dollar toward the better deal would raise utility, so the consumer isn't done until they're equal.

Condition	Implication
$MRS_{12} > p_1/p_2$	Consumer values good 1 above its market rate → buy more x_1 , less x_2 .
$MRS_{12} < p_1/p_2$	Consumer values good 1 below its market rate → buy less x_1 , more x_2 .
$MRS_{12} = p_1/p_2$	Optimal — no reallocation can raise utility.

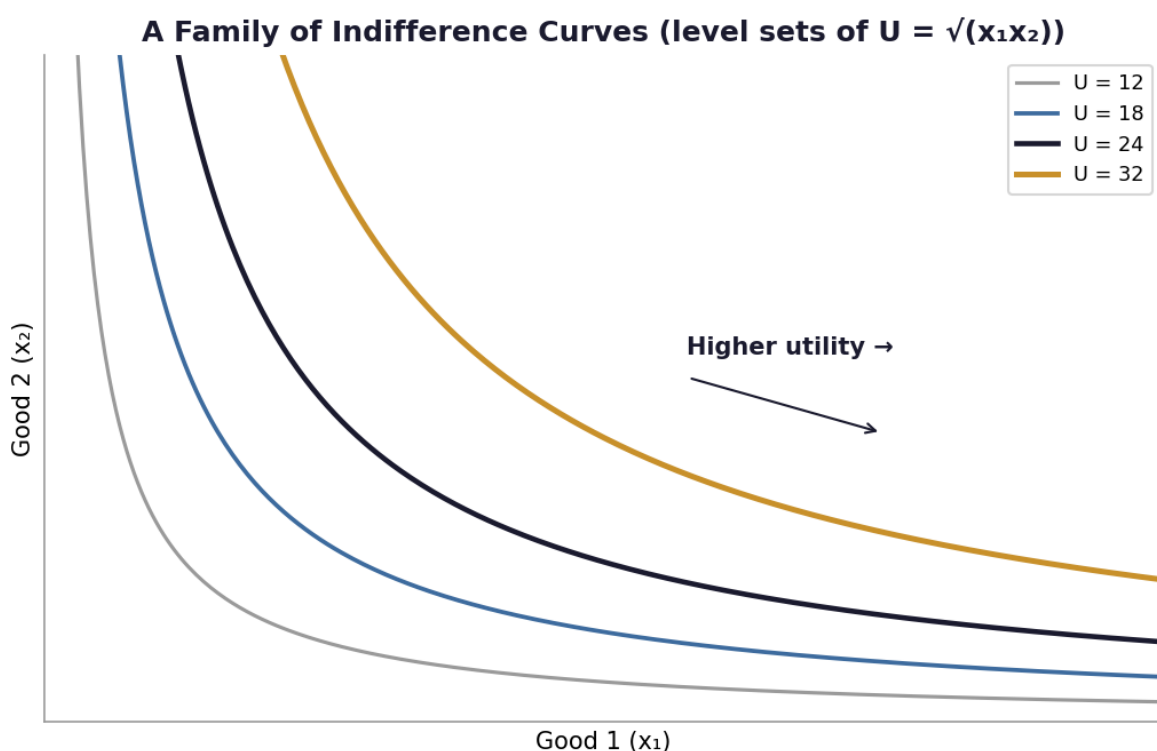


Figure 1.3 — A family of indifference curves for $U = \sqrt{(x_1x_2)}$. The consumer's goal: reach the highest curve the budget allows.

7. Worked Example — Cobb–Douglas, Start to Finish

Problem

$U(x_1, x_2) = x_1^{1/2}x_2^{1/2}$, $p_1 = 2$, $p_2 = 4$, $m = 120$. Find (x_1^*, x_2^*) .

Step 1 — MRS

$MU_1 = \frac{1}{2}x_1^{-1/2}x_2^{1/2}$; $MU_2 = \frac{1}{2}x_1^{1/2}x_2^{-1/2}$. The halves and roots cancel in the ratio: **MRS = x_2/x_1** .

Step 2 — Tangency

$x_2/x_1 = p_1/p_2 = 2/4 \rightarrow x_2 = \frac{1}{2}x_1$.

Step 3 — Substitute into the budget

$2x_1 + 4(\frac{1}{2}x_1) = 120 \rightarrow 4x_1 = 120 \rightarrow x_1^* = 30, x_2^* = 15$.

Verification and the Expenditure-Share Shortcut

Budget: $2(30) + 4(15) = 120 \checkmark$. MRS: $15/30 = \frac{1}{2} = p_1/p_2 \checkmark$.

Shortcut: for $U = x_1^\alpha x_2^\beta$, the consumer spends fraction $\alpha/(\alpha+\beta)$ of income on good 1 — **exponents are expenditure shares**. Here: $\frac{1}{2} \times 120 = \$60$ on each good $\rightarrow x_1^* = 60/2 = 30$, $x_2^* = 60/4 = 15$, in one line. Use the full method for marks; use the shortcut to check.

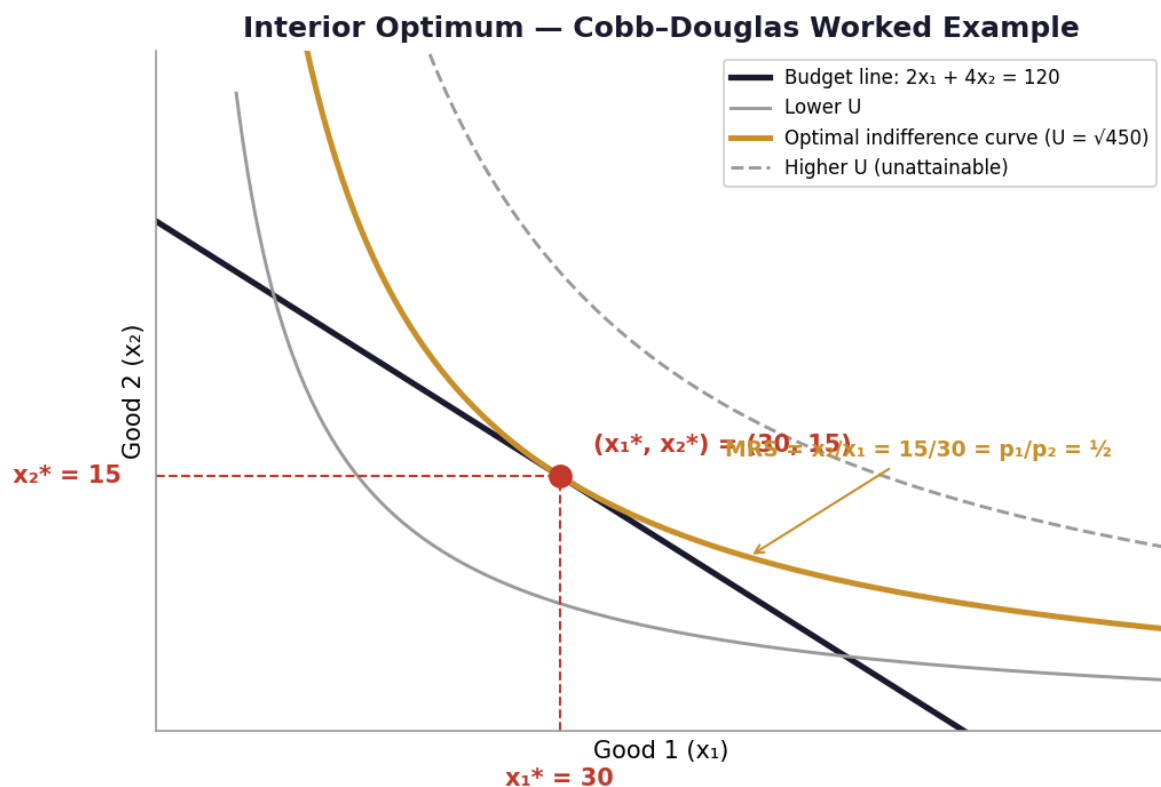


Figure 1.4 — The optimal bundle (30, 15): the budget line is tangent to the highest attainable indifference curve, where $MRS = p_1/p_2 = \frac{1}{2}$.

8. Worked Example — Perfect Complements at the Kink

Problem

$U = \min(2x_1, x_2)$, $p_1 = p_2 = 1$, $m = 30$. No tangency exists — the curves are kinked, so the MRS is undefined at the optimum.

Step 1 — Kink condition

At the optimum both arms of the min bind equally: $2x_1 = x_2$. (Buying more of one arm than the other wastes income on units that add no utility.)

Step 2 — Budget

$x_1 + x_2 = 30$ with $x_2 = 2x_1 \rightarrow 3x_1 = 30 \rightarrow x_1^* = 10, x_2^* = 20$.

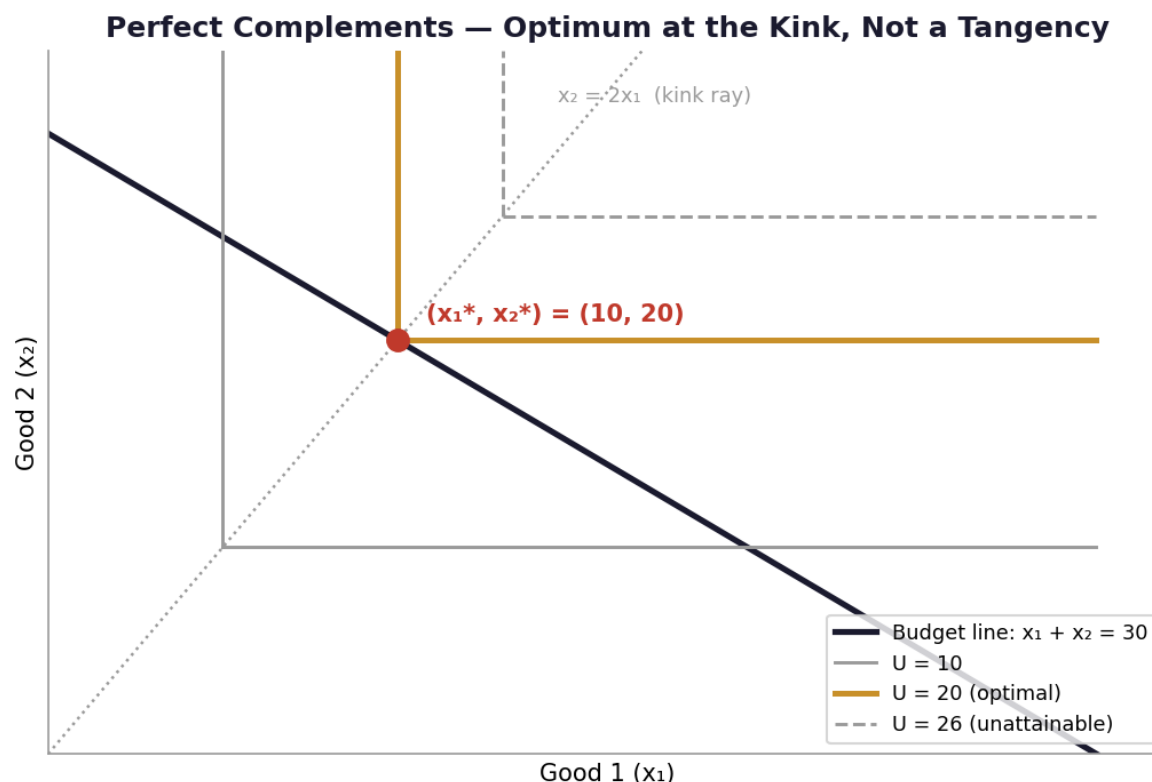


Figure 1.5 — Perfect complements: the optimum sits at the kink on the budget line, along the ray $x_2 = 2x_1$. No tangency is involved.

9. Corner Solutions

With perfect substitutes the MRS is constant, so no interior tangency exists — the consumer compares bang-per-buck once and goes all-in:

Condition	Corner solution
$MU_1/p_1 > MU_2/p_2$ everywhere (curves steeper than budget line)	All income on good 1: $(x_1^*, x_2^*) = (m/p_1, 0)$
$MU_1/p_1 < MU_2/p_2$ everywhere (curves flatter than budget line)	All income on good 2: $(x_1^*, x_2^*) = (0, m/p_2)$

Example: $U = 3x_1 + 2x_2$ with $p_1 = 2$, $p_2 = 1$, $m = 20$. Bang-per-buck: $3/2 = 1.5$ vs $2/1 = 2$ — good 2 wins everywhere, so the optimum is $(0, 20)$. Applying the tangency condition here ($3/2 = 2$, false) yields nonsense; recognising the functional form first is what saves you. Using tangency on substitutes or complements is the most common lost mark across Weeks 1–3 problem sets.

10. Key Propositions

Proposition	Statement	Implication
Utility representation	Well-behaved preferences are representable by an ordinal $U(x_1, x_2)$.	Any monotonic transformation of U gives the same curves and the same demand.
MRS formula	$MRS_{12} = MU_1/MU_2 = \text{slope} $ of the indifference curve.	Links the utility function's calculus to willingness to trade.
Interior optimum	$MRS_{12} = p_1/p_2$ and the budget holds with equality.	Equal bang-per-buck across goods; two equations pin down the bundle.
Corner/kink optima	Substitutes $\rightarrow$ all-in on the better bang-per-buck; complements $\rightarrow$ kink condition $ax_1 = bx_2$ plus budget.	Tangency is a method for smooth convex preferences only.
Cobb–Douglas shares	With $U = x_1^\alpha x_2^\beta$, spending on good 1 is $\alpha/(\alpha+\beta)$ of income.	The fastest sanity check in consumer theory.

11. Common Student Mistakes

- **Treating utility as cardinal.** $U = 10$ is not “twice as good” as $U = 5$ — only rankings matter, which is also why transformed utility functions give identical answers.
- **Using tangency on the wrong functional form.** Straight lines (substitutes) and kinks (complements) have no interior tangency — identify the form before choosing the method.
- **Forgetting the second equation.** $MRS = p_1/p_2$ alone is one equation in two unknowns; the optimum also requires the bundle to exhaust the budget.
- **Not simplifying the MRS before substituting.** Reduce MU_1/MU_2 symbolically first — it prevents arithmetic slips and earns method marks.
- **Mixing up the two slopes.** Budget-line slope = $-p_1/p_2$ (prices; the market's trade-off). Indifference-curve slope = $-MRS$ (preferences; the consumer's). They are only equal at an interior optimum.
- **Generic Cobb–Douglas MRS errors.** For $U = x_1^\alpha x_2^\beta$, $MRS = (\alpha/\beta)(x_2/x_1)$ — it is x_2/x_1 only when $\alpha = \beta$. Differentiate carefully or log-transform first.

12. Practice Problems

Attempt before your first tutorial. Solutions provided separately.

- **Q1.** Show that $U = x_1x_2$ and $V = x_1^2x_2^2$ represent the same preferences, and state in one sentence why either earns full marks in an optimisation question.
- **Q2.** For $U = x_1^2x_2$, derive the MRS, evaluate it at $(2, 5)$, and show it is diminishing in x_1 along an indifference curve.
- **Q3.** $U = x_1^{0.25}x_2^{0.75}$, $m = 200$, $p_1 = 5$, $p_2 = 5$. Find the optimal bundle, then verify it with the expenditure-share shortcut.
- **Q4.** A consumer drinks coffee only with exactly two sugars. Write a utility function for these preferences and find the optimum at $m = 12$, $p_{\text{coffee}} = 3$, $p_{\text{sugar}} = 0.5$.
- **Q5.** $U = 3x_1 + 2x_2$, $p_1 = 2$, $p_2 = 1$, $m = 20$. Find the optimal bundle — and explain precisely why the tangency condition fails here.
- **Q6.** $U = \ln x_1 + x_2$, $p_1 = 1$, $p_2 = 2$, $m = 10$. Solve for the optimum. Then identify what is odd about the demand for x_1 — which variable does it not depend on? (This quasilinear quirk returns in welfare analysis.)

Next — Week 2: Demand and Comparative Statics: how the optimal bundle responds as income and prices move — demand functions, Engel curves, and the first look at income vs substitution effects.

Q1. Show $U = x_1x_2$ and $V = x_1^2x_2^2$ represent the same preferences.

$V = (x_1x_2)^2$ is a **monotonic (increasing) transformation** of U — squaring a positive number preserves order, so whenever U ranks bundle A above B , V does too.
 Identical rankings $\Rightarrow$ identical indifference curves $\Rightarrow$ identical demand. Either earns full marks because utility is **ordinal**: only the ranking carries meaning, never the numerical size.

Q2. For $U = x_1^2x_2$, derive the MRS, evaluate at $(2, 5)$, and show it diminishes in x_1 .

$MU_1 = 2x_1x_2$, $MU_2 = x_1^2$. $MRS = 2x_1x_2 \div x_1^2 = 2x_2/x_1$.
 At $(2, 5)$: $MRS = 2(5)/2 = 5$.
 Diminishing: along an indifference curve, moving right raises x_1 and lowers x_2 , so $2x_2/x_1$ **falls** — the curve flattens, i.e. it is convex. $\checkmark$

Q3. $U = x_1^{0.25}x_2^{0.75}$, $m = 200$, $p_1 = 5$, $p_2 = 5$. Optimal bundle; verify with expenditure shares.

Tangency: $MRS = (0.25/0.75)(x_2/x_1) = p_1/p_2 = 1$, so $x_2/x_1 = 3$, i.e. $x_2 = 3x_1$.
 Budget: $5x_1 + 5(3x_1) = 200 \rightarrow 20x_1 = 200 \rightarrow x_1^* = 10$, $x_2^* = 30$.
 Share check: good 1 gets $\alpha/(\alpha+\beta) = 0.25$ of income = \$50 $\rightarrow x_1 = 50/5 = 10 \checkmark$; good 2 gets \$150 $\rightarrow x_2 = 30 \checkmark$.

Q4. Coffee drunk only with exactly two sugars. Utility function; optimum at $m = 12$, $p_{\text{coffee}} = 3$, $p_{\text{sugar}} = 0.5$.

Fixed proportions (2 sugar per coffee) $\Rightarrow$ **perfect complements**: $U = \min(x_c, x_s/2)$.
 No tangency — use the proportion condition $x_s = 2x_c$.
 Budget: $3x_c + 0.5(2x_c) = 12 \rightarrow 4x_c = 12 \rightarrow x_c^* = 3$ coffees, $x_s^* = 6$ sugars.
 Check: $3(3) + 0.5(6) = 12 \checkmark$.

Q5. $U = 3x_1 + 2x_2$, $p_1 = 2$, $p_2 = 1$, $m = 20$. Optimum; why tangency fails.

Perfect substitutes — compare bang-per-buck: $MU_1/p_1 = 3/2 = 1.5$ vs $MU_2/p_2 = 2/1 = 2$.
 Good 2 wins *everywhere*.
 Corner solution: spend all income on good 2 $\rightarrow (x_1^*, x_2^*) = (0, 20)$.
 Tangency fails because $MRS = 3/2$ is **constant** and never equals $p_1/p_2 = 2$ — the straight indifference curves are never parallel to the budget line, so no interior tangency exists.

Q6. $U = \ln x_1 + x_2$, $p_1 = 1$, $p_2 = 2$, $m = 10$. Optimum; which variable does x_1 not depend on?

$MU_1 = 1/x_1$, $MU_2 = 1$. $MRS = 1/x_1 = p_1/p_2 = 1/2 \rightarrow \mathbf{x_1^* = 2}$.

Budget: $1(2) + 2x_2 = 10 \rightarrow \mathbf{x_2^* = 4}$.

$x_1^* = p_2/p_1$ depends only on the **price ratio — not on income m** . This is the quasilinear signature: all income effects fall on good 2. It returns in welfare analysis, where it makes consumer surplus an exact welfare measure.