

MATHS REFRESHER

Derivatives for Economics

Who this is for: students starting intermediate micro/finance who are rusty on differentiation.

1. What a Derivative Is

The derivative $f'(x)$ is the **slope of the function at a point** — the rate f changes per one-unit change in x , right there. That's the entire concept. Economics runs on it because every “marginal” anything (marginal utility, marginal cost, marginal product) is just a derivative: “how much does the total change if I add one more unit?”

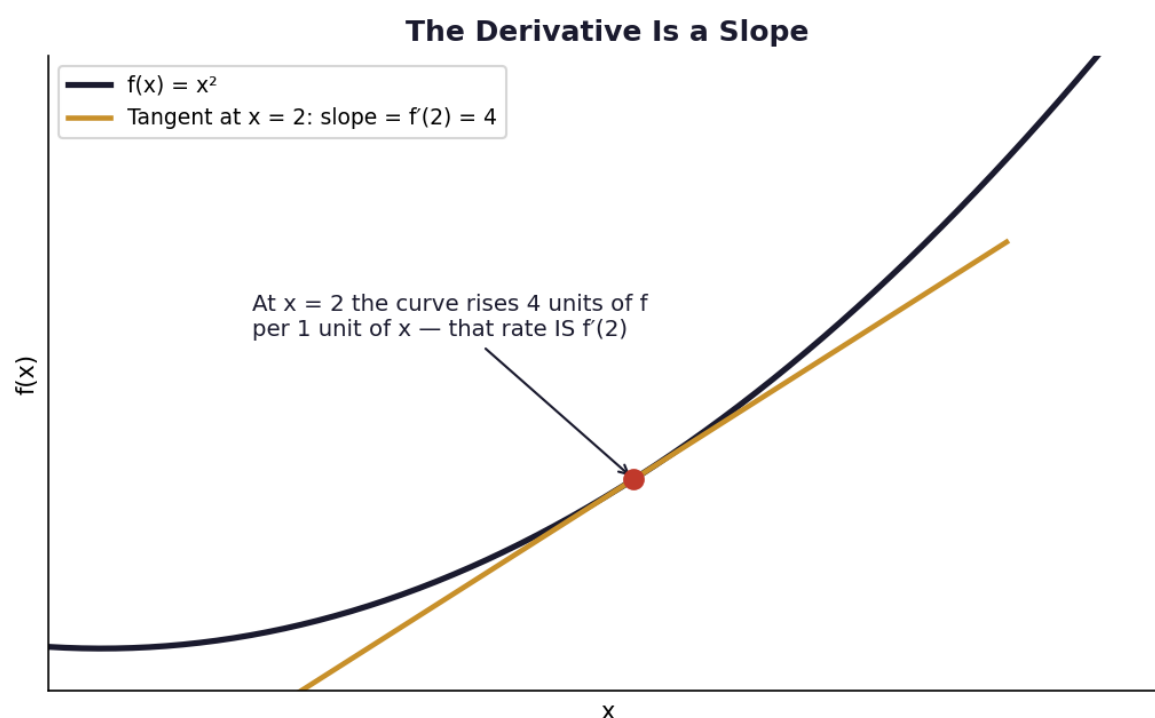


Figure 1 — $f(x) = x^2$ with its tangent at $x = 2$. The tangent's slope, 4, is the derivative there: $f'(x) = 2x$, so $f'(2) = 4$.

2. The Five Rules You Need

Rule	Statement	Example
Power rule	$d/dx (x^n) = n \cdot x^{n-1}$	$d/dx (x^3) = 3x^2$
Constant multiple	Constants ride along in front.	$d/dx (4x^2) = 8x$
Sum rule	Differentiate term by term.	$d/dx (x^2 + 3x) = 2x + 3$
Constants vanish	$d/dx (c) = 0$	$d/dx (7) = 0$
Natural log	$d/dx (\ln x) = 1/x$	$d/dx (5 \ln x) = 5/x$

The Rewriting Trick

The power rule handles fractions and roots **once you rewrite them as powers**:

$$1/x = x^{-1} \rightarrow d/dx = -x^{-2} \quad \sqrt{x} = x^{1/2} \rightarrow d/dx = \frac{1}{2}x^{-1/2}$$

Two habits: rewrite *before* differentiating, and remember the exponent **drops by one even when negative** (-1 becomes -2 , not 0). Nearly every differentiation slip in consumer theory is one of these two.

Chain Rule

$$d/dx [f(g(x))] = f'(g(x)) \cdot g'(x)$$

Differentiate the **outside** function leaving the inside alone, then multiply by the **inside's** derivative. Example: $d/dx (2x + 1)^3 = 3(2x + 1)^2 \times 2 = 6(2x + 1)^2$. In ECON2101 you'll mostly meet it inside logs: $d/dx \ln(2x) = (1/2x) \times 2 = 1/x$.

3. Partial Derivatives

With two variables, $U(x_1, x_2)$, a **partial derivative** differentiates with respect to one variable while **treating the other as a constant number**. Notation: $\partial U / \partial x_1$ (curly ∂ instead of d). That's the entire difference — the five rules above still do all the work.

Calculation — Partial of $U = x_1^\alpha x_2^\beta$

$\partial U / \partial x_1$: treat x_2^β as a constant riding along in front $\rightarrow \alpha x_1^{\alpha-1} x_2^\beta$.

$\partial U / \partial x_2$: now x_1^α is the constant $\rightarrow \beta x_1^\alpha x_2^{\beta-1}$.

Concrete case, $U = x_1^{0.5} x_2^{0.5}$: $MU_1 = 0.5 x_1^{-0.5} x_2^{0.5}$ and $MU_2 = 0.5 x_1^{0.5} x_2^{-0.5}$. Dividing them, everything cancels except **MRS** = x_2/x_1 — which is exactly the calculation in Week 1's worked example. This refresher and that chapter meet right here.

The Log Shortcut (why transformations matter)

Because utility is ordinal, you may replace $U = x_1^\alpha x_2^\beta$ with $\ln U = \alpha \ln x_1 + \beta \ln x_2$ before differentiating. Then $MU_1 = \alpha/x_1$ and $MU_2 = \beta/x_2$ — no negative exponents anywhere — and $MRS = (\alpha/\beta)(x_2/x_1)$ drops out in one line. If exponent algebra is the thing slowing you down, this is the workaround examiners fully accept.

4. Common Mistakes

- **Forgetting to drop the exponent by one** ($x^3 \rightarrow 3x^3$ instead of $3x^2$), especially with negative exponents ($x^{-1} \rightarrow -x^{-2}$, not $-x^0$).
- **Differentiating the constant away wrongly:** constants *multiplying* a term survive ($4x^2 \rightarrow 8x$); constants *added* vanish ($x^2 + 7 \rightarrow 2x$).
- **Treating the other variable as a variable** in a partial derivative — in $\partial U/\partial x_1$, the symbol x_2 is just a number.
- **Skipping the rewrite step** and trying to differentiate $1/x$ or $\sqrt{x}$ by instinct
- **Chain-rule amnesia inside logs:** $d/dx \ln(2x)$ needs the inner derivative (answer $1/x$, via $(1/2x) \times 2$), though note here it happens to equal $d/dx \ln x$ — because $\ln(2x) = \ln 2 + \ln x$.

5. Practice

- **Q1.** $d/dx (5x^3)$
- **Q2.** $d/dx (1/x^2)$
- **Q3.** $d/dx (\sqrt{x})$
- **Q4.** $d/dx (\ln x + 3x)$
- **Q5.** Both partial derivatives of $U = x_1^{0.5} x_2^{0.5}$
- **Q6.** Both partials of $U = x_1^2 x_2$, and the MRS. (You've seen this answer in Week 1.)

Answers

Q1: $15x^2$. Q2: $-2x^{-3} = -2/x^3$. Q3: $\frac{1}{2}x^{-1/2} = 1/(2\sqrt{x})$. Q4: $1/x + 3$.
 Q5: $MU_1 = 0.5x_1^{-0.5}x_2^{0.5}$, $MU_2 = 0.5x_1^{0.5}x_2^{-0.5}$. Q6: $MU_1 = 2x_1x_2$, $MU_2 = x_1^2$, $MRS = 2x_2/x_1$.